Common Problems in Research (CPR)

Leonel Martinez

August 9, 2024

1

We frequently use the Lorentz invariant Mandelstam variables s, t, and u when discussing two body interactions such as $a + b \to c + d$. Show that

$$s + t + u \equiv m_a^2 + m_b^2 + m_c^2 + m_d^2$$

This is why for a given reaction, only two of the three Mandelstam variables can be considered truly independently. The third one can be uniquely defined as a function of the other two.

2

In our research, we frequently encounter tasks of selecting a pair of decay particles from more than one possible combinations. For example, how to choose 2 photons coming from a π^0, if more than two photons are identified in the same event. Or how do we choose two protons (and an antiproton is expected to be the missing particles) if more than two protons are detected in the same event? Can we select the photons based on their invariant mass being closer to π^0 (or choose two protons if the corresponding missing mass is closer to antiproton)?

We can generalize this into a simple statistics problem:

Given two uncorrelated variables x_1 and x_2, and assuming both are evenly distributed between the lower bound a and the upper bound b. The probability distributions for both variables is given (obviously) by $f(x_{1,2}) = \frac{1}{b-a}$. From each pair of x_1 and x_2, choose the one closest to the midpoint $c = \frac{b+a}{2}$.

a.) Show that the distribution of the chosen value x_f is given by

$$f(x_f) = \begin{cases} \frac{4(x_f - a)}{(c-a)^2} & a \leq x_f < c \\ \frac{4(b - x_f)}{(b-c)^2} & c < x_f \leq b \end{cases}$$

which can also be written as

$$f(x_f) = \frac{\left(\frac{b-a}{2} - \left|\frac{b+a}{2} - x\right|\right)}{(b-a)^2}$$

Verify that $\int_a^b f(x_f) dx_f = 1$

b.) Generalize this problem to 3 uncorrelated variables x_1, x_2, x_3 also evenly distributed between a and b, and we choose the closest one to the midpoint and call that as x_f. Then $f(x_f) =?$. What if there are n uncorrelated variables to choose from?

c.) For $n = 2, 3$, what if the anchor point is not the midpoint, but an arbitrary point between a and b. What will the PDFs become?

d.) What did we learn from this exercise?

3

The cascade baryon (Ξ) gets its name for the various particles that are produced in steps. The baryon undergoes rapid decays into stabler and lighter particles, best characterized as a "cascade" of decays (Physicists are very creative!).

a.) What is the main decay channel of both Ξ baryons? If there are any subsequent decays provide them as well.

b.) By what mechanism do the Ξ baryons decay by? Are any conservation laws broken? What does it mean?

c.) What is the quark content of the Ξ baryons and their main subsequent decay channels?

d.) The Ξ baryons have two possible charges, using their charge and quark content determine the fractional charge of the up, down, and strange quark.